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THE INFLUENCE OF THE LOAD AND JOINT CARTILAGE ELASTICITY MODULES ON SYNOVIAL FLUID VISCOSITY

WPŁYW OBCIĄŻENIA I MODUŁÓW SPRĘŻYSTOŚCI CHRZĄSTKI STAWOWEJ NA LEPKOść CIECZY SYNOWIALNEJ

Key words:
joint cartilage, superficial layer, hypo-hyper-elasticity modules, non-Newtonian synovial fluid, viscosity changes, hydrodynamic lubrication, rotation, squeezing, EHD-deductions

Summary
During classical journal bearing lubrication the lubricant viscosity is independent of physical properties of cooperating bodies, which is well known

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by virtue of Hersey-Stribec (H-S) curve presenting friction coefficient vs. Hersey number = viscosity x velocity/pressure.

The result obtained by the H-S is valid for two cooperating bodies with homogeneous, isotropic properties, and for Newtonian oils omitting the elasto-hydrodynamic effects. In the presented paper, we take into account the two cooperating human joint cartilage surfaces, which, after new AFM measurements, have non-homogeneous hypo- or hyper-elastic properties, and the synovial fluid has non-Newtonian features. Moreover, the cartilage surface during human limb motion and during the squeezing and boosted squeezing effects gains important small deformations. From the abovementioned description, it follows that the H-S result cannot be acceptable in human joint lubrication [L. 1–6].

During human joint hydrodynamic lubrication, we observe the influence of the material coefficients of the hypo- and hyper-elastic cartilage tissue on the apparent viscosity of non-Newtonian synovial fluid occupying the thin joint gap limited by the two cartilage superficial layers. This problem has not been considered in scientific papers describing the hydrodynamic lubrication of the human joint. This problem attains significant meaning because, after numerous AFM laboratory measurements confirmed by the literature achievements, it follows that the joint cartilage tissue with a thin polar membrane made of two lipid molecules has no isotropic but anisotropic properties in general. These membranes are flat sheets that form a continuous barrier around the cartilage cells. Non-homogeneous, anisotropic biological bodies as distinct from classical isotropic materials have the various values of elasticity, hypo-elasticity, or hyper-elasticity modules on individual places and directions. These places, loaded by the same forces, tend to various displacements and strains. In consequence, mutually connected physical implications caused by virtue of synovial fluid flow velocity and shear rates changes, indicate to us the conclusion that the dynamic viscosity of synovial fluid gains value variations caused by the cartilage’s physical properties during human joint lubrication.

ABOUT NON-CLASSICAL JOINT CARTILAGE FEATURES

The necessity and rightness of the solutions of the presented problem illustrated in this paper have been initiated and inaugurated by the interesting data of physical and chemical properties of the joint cartilage and its superficial layer of synovial fluid obtained from AFM measurements performed after 2004 in micro and nano level [L. 7–8].

EU grants MTKD-CT-517226 and IRSES, 612593 were used to prove the hypothesis that the anisotropic or, in particular, hypo-and hyper-elastic, human joint cartilage with elasticity parameters in range of 12 to 50 MPa have an influence on the apparent viscosity of non-Newtonian synovial fluid in the interval from 0.003 to 0.3000 Pas [L. 9].
Human joint cartilage considered as the hypo-elastic specimen has the following properties:

The cartilage as a solid has a preferred shape.

Cartilage deformations are reversible, i.e. removing the loads implies returning to the original shape.

The strain in cartilage depends only on the stress.

The stress is a nonlinear function of strain, even when the strains are small [L. 8–10].

We have a simple isotropic and complex anisotropic hypo-elastic cartilage model. A simple isotropic model assumes that the response of a cartilage is independent of its orientation with respect to the loading direction. In principle, it would be possible to develop complex, anisotropic, hyper-elasticity models, but this is rarely done. J.J. Garcia et al. had already considered the combined biphasic cartilage model based on transversely isotropic hypo-elastic features [L. 11].

After J.J. Garcia, the joint gap height had depended on the stress occurring in the cartilage’s superficial layer and on the deformations of the hypo-elastic cartilage, which determined the effective Young modulus and effective stress in hypo-elastic cartilage under rapid load. The human joint cartilage had been assumed transversely isotropic with the plane of isotropy (1-2), parallel to the surface of the cartilage and perpendicular to the loading axis (3). J.J. Garcia had considered two mechanical constants, namely the Young modulus $E_1$ in the plane of isotropy, and the Young modulus $E_3$ that is normal to the plane of isotropy, i.e. in the load line direction [L. 11].

Cartilage soft tissues are naturally made of a matrix and fibres that present some privileged directions. The mechanical behaviour of these cartilage tissues highlights different phenomena as hysteretic, stress softening, or relaxation. The hyper-elastic constitutive equation is typically the basis of the model that describes the behaviour of the material. The hyper-elastic constitutive equations can be isotropic or anisotropic. It is generally expressed by means of strain components or strain invariants.

Based on the performed measurements, it had been stated that synovial fluid lubrication flow in the human-joint gap idiopathically generates in the non-homogeneous superficial layer for which the hyper-elastic modulus does not have the same values at each place [L. 12–15].

EXPERIMENT

In the performed experiments we took into account two separate places laying on the external sheet of the superficial layer, having the same hyper-elastic modules but loaded with the various forces or have various values of hyper elastic modules and are loaded by the same force values.
Lubrication by rotation

At first, we considered the gap joint limited by the two cooperating joint cartilage surfaces lubricated by rotation, where the lower surface is loaded unilateral by the force $P_1$ at point 1 and by the force $P_2$ at point 2.

On the external cartilage surface, we considered two points to be in the neighbourhood and denoted by numbers 1 and 2. We assume that at points 1 and 2, the joint cartilage has hyper-elasticity modules $E_{x1}$, $E_{x2}$, where $E_{x1} > E_{x2}$, which denotes that, at point 2, the cartilage is more flexible and biodegradable than the cartilage at point 1. We assume that the cartilage points 1 and 2 are loaded by the forces $P_1$, $P_2$, respectively, where $P_1 = P_2$. This situation is presented in Fig. 1.

![Fig. 1. Two points 1 & 2 on the cartilage superficial layer: a) for elasticity modules $E_{x1}>E_{x2}$, loaded by the same forces $P_1$, $P_2$, $P_1 = P_2$, b) equivalent load model for $E_{x1} = E_{x2}$, $P_1<P_2$.](image)

Rys. 1. Dwa miejsca 1 i 2 na warstwie wierzchniej chrząstki stawowej: a) obciążone siłami $P_1$, $P_2$, $P_1 = P_2$ dla modułów sprężystości $E_{x1}>E_{x2}$, b) model równoważny: siły $P_1<P_2$, dla modułów sprężystości $E_{x1} = E_{x2}$

Fig. 2 illustrates the cartilage surface displacements implied from the load system presented in Fig. 1.

![Fig. 2. Cartilage surface displacements $u_1$, $u_2$ of two points 1 and 2](image)

Rys. 2. Ugięcia powierzchni chrząstki $u_1$, $u_2$ w punktach 1 i 2
The comments are as follows: The displacement $u_1$ of cartilage at point 1 is less than the cartilage displacement $u_2$ at point 2, $(u_1 < u_2)$, because the cartilage at point 2 is more flexibility than at point 1 $(E_{x1} > E_{x2})$ and $P_1 = P_2$. Displacements “$u$” are calculated from geometrical equations of hyper-elasticity theory [L. 10].

Fig. 3 illustrates the total gap height of the considered joint implied from the load system presented in Fig. 1 and from geometrical displacements given in Fig. 2.

We have the following comments: Total gap height $\varepsilon_{T1}$ at point 1 is less than total gap height $\varepsilon_{T2}$ at point 2, i.e. $(\varepsilon_{T1} < \varepsilon_{T2})$, because the total gap height denotes the primary gap height plus displacement depicted in Fig. 2.

Fig 4 shows the velocity distributions across the gap height at two points by virtue of the total gap height presented in Fig. 3.

Necessary Comments: Velocity values are calculated using J.L. Poisseuille’s assumption [L. 16]. For stationary flow, it follows that the flow rates at points 1 and 2 are identical despite the fact that the bore of the gap at point 1 is less than the bore at point 2. From Hagen-Poiseuille’s Law, it follows that, in this case, the values of velocity distribution $v_2$ are less than values $v_1$, i.e. $(v_1 > v_2)$ [L. 16].
Fig 5 shows the implication of shear rate values existing at points 1 and 2 and the corresponding dynamic viscosity values of synovial fluid during lubrication based on the velocity magnitudes depicted in Fig. 4.

![Diagram of shear rate and viscosity values](image)

Fig. 5. The implication from the inequality of shear rate values $\Theta_1 > \Theta_2$ at points 1 and 2 to the corresponding inequality of dynamic viscosities values: $\eta_1 < \eta_2$

Rys. 4. Implikacja od nierówności dla prędkości ścimania $\Theta_1 > \Theta_2$ w punktach 1, 2 do odpowiadającej im nierówności wartości lepkości dynamicznych cieczy $\eta_1 < \eta_2$.

Necessary finally Comments: The results of calculations are obtained by virtue of Reiner Rivlin’s model for constitutive strain-stress relationship for pseudo-plastic fluids in thin layer and based on D. Dowson’s and our own measurements of dependences $\eta$ vs $\Theta$ performed for non-Newtonian synovial fluids [L. 9]. From the abovementioned Laws and experiments, it follows that the inequality of velocity $v_2$ being less than velocity $v_1$, i.e. $(v_1 > v_2)$ (see Fig. 4) implies less shear rate $\Theta_2$ than shear rate $\Theta_1$, i.e. $(\Theta_1 > \Theta_2)$ (see Fig. 5). Hence, dynamic viscosity $\eta_1$ at point 1 is less than dynamic viscosity $\eta_2$ at point 2, i.e. $(\eta_1 < \eta_2)$.

The final result of the above sequences of implications indicates the following expression:

$$(E_X1 > E_X2) \rightarrow (u_1 < u_2) \rightarrow (\epsilon_{T1} < \epsilon_{T2}) \rightarrow (v_1 > v_2) \rightarrow (\Theta_1 > \Theta_2) \rightarrow (\eta_1 < \eta_2). \quad (1)$$

Lubrication by squeezing

The gap joint is limited by the two cooperating joint cartilage surfaces lubricated by squeezing, where the upper and lower surfaces are loaded unilateral by the force $P_1, (-P_1)$ at point 1 and by the force $P_2, (-P_2)$ at point 2.

Fig 6 shows two points to be in neighbourhood and denoted by number 1 and 2 on the external upper and lower cartilage surfaces.
Fig. 6. Two places 1 & 2 on the cartilage superficial layer: a) with larger (2) and smaller (1) flexibility for larger $E_{x1}$ and smaller $E_{x2}$ elasticity modules ($E_{x1}>E_{x2}$), loaded by the same forces $P_1 = P_2$ and directed at each point in the opposite direction in accordance to the assumed hydrodynamic squeezing lubrication of the bone head of human joint, b) equivalent load model for the same values of elasticity modules.

Rys. 6. Dwa miejsca 1 i 2 na warstwie wierzchniej chrząstki stawowej: a) o mniejszej (większej) podatności z większym (mniejszym) $E_{x1}$($E_{x2}$) modulem nad sprężystości ($E_{x1}>E_{x2}$) obciążone równymi siłami $P_1 = P_2$ skierowanymi przeciwieś stosownie do przyjętego smarowania stawu poprzez wyciskanie, b) model ekwiwalentnego obciążenia

Comments: We assume that at points 1 and 2, the joint cartilage has hyper-elasticity modules $E_{x1}$, $E_{x2}$, where $E_{x1}>E_{x2}$, denotes that, at point 2, the cartilage is more flexible and biodegradable than the cartilage at point 1. We assume that the cartilage loaded forces $P_1$, (-$P_1$); $P_2$, (-$P_2$) at points 1 and 2, respectively, have the same absolute values, i.e. $P_1$=$P_2$, but force $P$ has the opposite direction in relation to force $-P$.

Fig. 7 illustrates the cartilage surface displacements implied from the load system presented in Fig. 6.

Fig. 7. Cartilage surface displacements $u_1, u_2$ of two points 1 and 2

Rys. 7. Ugięcia powierzchni chrząstki $u_1, u_2$ w punktach 1 i 2

The comments are as follows: The sum of displacements $u_1$ of cartilage at point 1 is less than the sum of cartilage displacement $u_2$ at point 2, ($u_1<u_2$), because $P_1 = P_2$ and the cartilage at point 2 is more flexibility than at point 1. Displacements “$u$” are obtained from geometrical equations of hyper-elasticity theory [L. 10].
Fig. 8 illustrates the total gap height of the considered joint implied from the load system presented in Fig. 6, and from geometrical displacements given in Fig. 7.

Fig. 8. Gap height values $\varepsilon_{T1}$, $\varepsilon_{T2}$ at two points (1 and 2) in the considered joint
Rys. 8. Wysokości szczeliny $\varepsilon_{T1}$, $\varepsilon_{T2}$ dwóch punktów 1 oraz 2 na powierzchni stawu

Comments: Total gap height $\varepsilon_{T1}$ at point 1 is larger than total gap height $\varepsilon_{T2}$ at point 2, i.e. ($\varepsilon_{T1} > \varepsilon_{T2}$), because total gap height denotes primary gap height minus the sum of the displacement for each point depicted in Fig. 7.

Fig. 9 shows the velocity distributions across the gap height at two points by virtue of total gap height presented in Fig. 8.

Fig. 9. Synovial fluid velocity distributions: $v_1$, $v_2$ across the gap height at points 1, 2
Rys. 9. Rozkłady prędkości: $v_1$, $v_2$ cieczy synowialnej w punktach 1, 2

Necessary Comments: Velocity values are calculated based on J.L. Poisseuille’s assumption [L. 16]. For stationary flow, it follows that the flow rates at points 1 and 2 are identical, despite the fact that the bore of the gap at point 1 is larger than the bore at point 2.

From Hagen-Poiseuille’s Law, it follows that, in this case, the values of velocity distribution $v_2$ are less than values $v_1$, i.e. ($v_1 < v_2$) [L. 16].

Fig 10 shows the implication of shear rate values existing at points 1 and 2 and the corresponding dynamic viscosity values of synovial fluid during lubrication based on the velocity magnitudes depicted in Fig. 9.
Fig. 10. The implication from shear rate values inequality $\Theta_1 < \Theta_2$ at points 1 and 2 to the corresponding dynamic viscosities inequality $\eta_1 > \eta_2$.

Rys. 10. Implikacja od nierówności prędkości ścinania $\Theta_1 < \Theta_2$ dla przepływu w punktach 1, 2 do odpowiadającej im nierówności lepkości dynamicznych cieczy $\eta_1 > \eta_2$.

Necessary final comments: The results of calculations are obtained by virtue of the Reiner Rivlin model for constitutive strain-stress relationship for pseudo-plastic fluids in a thin layer and based on D. Dowson’s and our own measurements of dependences $\eta$ vs $\Theta$ performed for non-Newtonian synovial fluids [L. 9]. From the abovementioned laws and experiments, it follows that velocity $v_2$ is larger than velocity $v_1$, i.e. ($v_1 < v_2$) (see Fig. 9), which implies the larger shear rate $\Theta_2$ than shear rate $\Theta_1$, i.e. ($\Theta_1 < \Theta_2$) (see Fig. 10). Therefore, the dynamic viscosity $\eta_1$ at point 1 is larger than dynamic viscosity $\eta_2$ at point 2, i.e. ($\eta_1 > \eta_2$).

$\ast$ The final result of the above sequences of implications indicates the following expression:

$$(E_{X1} > E_{X2}) \rightarrow (u_1 < u_2) \rightarrow (\varepsilon_{T1} > \varepsilon_{T2}) \rightarrow (v_1 < v_2) \rightarrow (\theta_1 < \theta_2) \rightarrow (\eta_1 > \eta_2). \quad (2)$$

The abovementioned results for lubrication by rotation and lubrication by squeezing describing the influence of cartilage elasticity modulus and load joint on the dynamic viscosity value of synovial fluid are illustrated in Tab. 1.
Tab. 1. The influence of the load and cartilage elasticity modulus on the synovial fluid viscosity in rotation and squeezing lubrication

The dynamic viscosity of the lubricant or dynamic viscosity of synovial fluid generally determines the efficacy and quality of the performed lubrication process. The determination of the dependencies between the elasticity modulus of the cartilage or elasticity modulus of endoprosthesis head and the values of synovial fluid gives new possibilities and new tools for designers and producers of new types of intelligent endoprosthesis to gain the control of the dynamic
viscosity of synovial fluid or to control the dynamic viscosity of other bio-
acceptable fluids.

CONCLUSIONS

1. The following assumptions were made: 1. Human joint cartilage has hyper-
or hypo-elastic properties. 2. Joint is lubricated by stationary rotation, using
non–Newtonian pseudo-plastic synovial fluid. 3. Two points, 1 and 2, are in
the neighbourhood and lying in the external cartilage surface and loaded by
the same forces. 4. Elasticity modulus $E_{x1}$ of cartilage at point 1 is larger
than elasticity modulus $E_{x2}$ at point 2, i.e. Cartilage at point 2 is more
flexibly than cartilage at point 1 ($E_{x1} > E_{x2}$). These assumptions lead to
the conclusion that dynamic viscosity $\eta_1$ of synovial fluid near point 1 is less
than dynamic viscosity $\eta_2$ of synovial fluid near point 2, i.e. ($\eta_1 < \eta_2$).

2. The following assumptions were also made assumptions: 1. Human joint
cartilage has hyper- or hypo-elastic properties. 2. The joint is lubricated by
the stationary squeezing using non–Newtonian pseudo-plastic synovial fluid.
3. Two points, 1 and 2, are in the neighbourhood and lying in the external
upper and lower cartilage surfaces loaded by the same force in opposite
directions at each point. 4. Elasticity modulus $E_{x1}$ of cartilage at point 1 is
larger than elasticity modulus $E_{x2}$ at point 2, i.e. cartilage at point 2 is more
flexible than cartilage at point 1 ($E_{x1} > E_{x2}$). These assumptions lead to
the conclusion that dynamic viscosity $\eta_1$ of the synovial fluid near point 1 is
larger than dynamic viscosity $\eta_2$ of the synovial fluid near point 2, i.e.
($\eta_1 > \eta_2$).

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Streszczenie

Model zmian współczynnika tarcia w zależności od liczby Herseya, czyli stosunku iloczynu lepkości i prędkości do ciśnienia, świadczy o niezależności lepkości czynnika smarującego od własności fizycznych smarowanych powierzchni w klasycznych łożyskach ślizgowych.

Jednak ważny rezultat Herseya-Stribecka H-S obowiązuje przede wszystkim dla dwóch jednorodnych, izotropowych współpracujących ciał oraz cieczy smarującej o newtonowskich właściwościach przy pominięciu elasto-hydro-dynamicznych efektów. Niniejsza praca dotyczy smarowania dwóch współpracujących powierzchni pokrytych chrząstką stawową, która według najnowszych pomiarów przeprowadzonych za pomocą
mikroskopu sił atomowych AFM ma zmienne, niejednorodne, anizotropowe, a w szczególności nadsprężyste właściwości uzależnione od położenia i kierunku. Ciecz synowialna jest nieneuwotonowska. Ponadto ruch kończyn człowieka, siadanie, klękanie, skoki powodują wzmoczone wyciskanie (boosted squeezing) cieczy synowialnej w warstwce chrząstki stawowej skutkujące chociaż małymi, ale jednak bardzo istotnymi niepomijalnymi deformacjami. Stąd wynika hipoteza o braku akceptacji rezultatów H-S w zakresie smarowania stawów. Podczas smarowania stawów człowieka prawie zawsze występuje problem wpływu modułów nadsprężystych chrząstki stawowej na lepkość pozorną nieneuwotonowskiej cieczy synowialnej zalegającej w szczelinie stawu. Według informacji autora rozpatrywanie tego ważnego problemu nie jest uwzględniane we współczesnej literaturze naukowej z zakresu hydrodynamiki stawów. Problem ten nabiera jednak szczególnie dużego znaczenia w świetle najnowszych własnych badań chrząstek stawowych w mikro- i nanoskalę potwierdzonych również w literaturze naukowej. Chrząstka stawowa pokryta dwuwarstwą fosfolipidów o różnym stopniu upakowania ma właściwości anizotropowe. Biologicznie niejednorodne ciała anizotropowe w odróżnieniu od klasycznych jednorodnych izotropowych materiałów mają różne wartości współczynników sprężystości i nadsprężystości w poszczególnych miejscach oraz kierunkach. Wskazane miejsca obciążone jednakowymi siłami odznaczają się wtedy różnymi przemieszczeniami i odkształczeniami. W konsekwencji opisanych w niniejszej pracy wzajemnych Implikacji związków fizycznych wywołanych zmianami prędkości przepływu oraz deformacji dochodzi się do wniosku, że lepkość dynamiczna cieczy synowialnej ulega zmianom w trakcie smarowania stawów w zależności od fizycznych właściwości chrząstki stawowej.